

A direct link between neutrinoless double beta decay and leptogenesis in a seesaw model with S_4 symmetry

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We study how leptogenesis can be implemented in a seesaw model with S_4 flavor symmetry, which leads to the neutrino tri-bimaximal mixing matrix and degenerate right-handed (RH) neutrino spectrum. Introducing a tiny soft S_4 symmetry breaking term in the RH neutrino mass matrix, we show that the flavored resonant leptogenesis can be successfully realized, which can lower the seesaw scale much so as to make it possible to probe in colliders. Even though such a tiny soft breaking term is essential for leptogenesis, it does not significantly affect the low energy observables. We also investigate how the effective light neutrino mass $|\langle m_{ee} \rangle|$ associated with neutrinoless double beta decay can be predicted along with the neutrino mass hierarchies by imposing experimental data of low-energy observables. We find a direct link between leptogenesis and neutrinoless double beta decay characterized by $|\langle m_{ee} \rangle|$ through a high energy CP phase ϕ , which is correlated with low energy Majorana CP phases. It is shown that our predictions of $|\langle m_{ee} \rangle|$ for some fixed parameters of high energy physics can be constrained by the current observation of baryon asymmetry.

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I. INTRODUCTION

Recent experiments of the neutrino oscillation go into a new phase of precise determination of mixing angles and mass squared differences [1], indicating that the tri-bimaximal (TBM) mixing for three flavors can be regarded as so-called PMNS mixing matrix $U_{\text{PMNS}} \equiv U_{\text{TB}} P_\nu$ in the lepton sector [2]

$$U_{\text{TB}} = \begin{pmatrix} \frac{\sqrt{2}}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}, \quad (1)$$

and P_ν is a diagonal matrix of phase for Majorana neutrinos. However, properties related to the leptonic CP violation are completely unknown yet. The large mixing angles, which may be suggestive of a flavor symmetry, are completely different from the quark mixing ones. In last few years there have been lots of efforts in searching for models which produce the TBM pattern for the neutrino mixing matrix, and a fascinating way seems to be the use of some discrete non-Abelian flavor groups added to the gauge groups of the Standard Model. There is a series of models based on the symmetry group A_4 [3], T' [4] and more recently S_4 [5–7].

In addition to the explanation for the smallness of neutrino masses, type-I seesaw model [8], in which heavy right-handed singlet Majorana neutrinos are introduced, has another appealing feature so-called leptogenesis mechanism for the generation of the observed baryon asymmetry of the Universe (BAU) through the decay of heavy Majorana neutrinos [9]. If this BAU originated from leptogenesis, then CP symmetry in the leptonic sector must be broken. So any observation of the leptonic CP violation, or demonstrating that CP is not a good symmetry of the leptons, can strengthen our belief in leptogenesis. For Majorana neutrinos there are two additional phases in U_{PMNS} , one (or a combination) of which in principle can be explored through neutrinoless double beta ($0\nu 2\beta$) decay [10]. Although the exact TBM mixing pattern forbids low energy CP violation measurable in neutrino oscillation due to $U_{e3} = 0$, it may allow CP violation due to the Majorana phases. Therefore, it is interesting to explore the existence of CP violation in the lepton sector due to the Majorana CP phases in the light of leptogenesis. Since $|\langle m_{ee} \rangle|$ depends on the Majorana CP phases, we examine if there exists a link between $0\nu 2\beta$ decay and BAU.

In the neutrino models with some discrete flavor symmetries, it is worthwhile to examine if leptogenesis can work out while keeping TBM pattern for neutrino mixing matrix. Motivated by this issue, in this letter, we study how leptogenesis can work in a seesaw model with S_4 flavor symmetry [11]. As anticipated, leptogenesis can not work in seesaw models with exact $SU(2)_L \times U(1)_Y \times S_4$ symmetry mainly due to the fact that the discrete symmetry S_4 leads to zero value of U_{e3} and thus could not

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generate lepton asymmetry. To make leptogenesis successfully realized, it is essential to break S_4 symmetry. In the case that the right-handed (RH) neutrinos are hierarchical in mass, successful leptogenesis requires that the RH neutrinos are superheavy, which makes inaccessible to colliders and gives rise to the overproduction of gravitinos during reheating in the supersymmetric scenarios [12]. These problems can be avoided if we consider an almost degenerate heavy RH neutrino mass spectrum so that lepton asymmetry can be resonantly generated [13, 14]. In this paper, we especially study how this so-called resonant leptogenesis can be implemented in a seesaw model with S_4 flavor symmetry [15]. In this respect, the model has to produce degenerate RH neutrino mass spectrum at the leading order. The S_4 seesaw model relevant to our purpose has been proposed in [7]. We examine how resonant leptogenesis can be implemented by introducing a small perturbation in heavy RH neutrino mass matrix M_R while keeping Dirac neutrino mass matrix and charged lepton mass matrix unchanged in the model [7]. We also show that leptogenesis can be linked to the neutrinoless double beta decay through seesaw mechanism with a small perturbation in M_R .

This work is organized as follows. In Sec. II, we study how low energy neutrino oscillation observables are predicted in a supersymmetric seesaw model based on the flavor symmetry group S_4 . We also discuss about the effective neutrino mass associated with neutrinoless double beta decay. In Sec. III, we examine how successful leptogenesis can be implemented by introducing a soft symmetry breaking term in the heavy RH neutrino mass matrix and show how leptogenesis can be linked to $0\nu 2\beta$ decay. Sec. IV is devoted to our conclusion.

II. LOW ENERGY OBSERVABLES

Although there have been several proposals to construct lepton mass matrices in the framework of seesaw

incorporating S_4 symmetry [6, 16], in this paper, we consider the model proposed in [7], which gives rise to TBM mixing pattern of the lepton mixing matrix [2] and leads to degenerate heavy RH neutrino mass spectrum which is essential for resonant leptogenesis. The model is supersymmetric and based on the flavor discrete group $G_f = S_4 \times Z_3 \times Z_4$, where the three factors play different roles. The reasons, why we consider supersymmetry, are to simplify the choice of desirable vacuum alignment, and low scale leptogenesis is well motivated in supersymmetry thanks to the gravitino problem. In general, there exists a contribution to leptogenesis via scalar right-handed sneutrinos, but their effect depends on soft susy breaking terms. In our study, we do not consider the contribution by simply assuming that the values of the soft susy breaking parameters do not give rise to the contribution to leptogenesis.

The S_4 component controls the mixing angles, the auxiliary Z_3 symmetry guarantees the misalignment in flavor space between the neutrino and the charged lepton mass eigenstates, and the Z_4 component is crucial to eliminating the unwanted couplings and reproducing the observed mass hierarchy. In this framework the mass hierarchies are controlled by the spontaneous breakdown of the flavor symmetry instead of the Froggatt-Nielsen mechanism [17]. S_4 is the discrete group given by the permutations of four objects and has been studied in literature [5], but with different aims and different results. It is composed by 24 elements, divided into 5 irreducible representations: two singlets, 1_1 and 1_2 , one doublet, 2 , and two triplets, 3_1 and 3_2 . The technical details of the group are shown in [7]. The multiplication rules between the various representations are as follows; $1_i \otimes 1_j = 1_{((i+j) \bmod 2)+1}$, $1_i \otimes 2 = 2$, $1_i \otimes 3_j = 3_{((i+j) \bmod 2)+1}$, $2 \otimes 2 = 1_1 \oplus 1_2 \oplus 2$, $2 \otimes 3_i = 3_1 \oplus 3_2$, $3_i \otimes 3_i = 1_1 \oplus 2 \oplus 3_1 \oplus 3_2$, $3_1 \otimes 3_2 = 1_2 \oplus 2 \oplus 3_2 \oplus 3_2$, with $i, j = 1, 2$. The matter fields in the lepton sector and the flavons under G_f of the model are assigned as Table I.

TABLE I: Representations of the matter fields in the lepton sector and the flavons under $S_4 \times Z_3 \times Z_4$. And ω is the third root of unity, i.e. $\omega = e^{i2\pi/3}$.

Field	l	e^c	μ^c	τ^c	ν^c	$h_{u,d}$	φ	χ	ϑ	η	ϕ	Δ
A_4	$\mathbf{3}_1$	$\mathbf{1}_1$	$\mathbf{1}_2$	$\mathbf{1}_1$	$\mathbf{3}_1$	$\mathbf{1}_1$	$\mathbf{3}_1$	$\mathbf{3}_2$	$\mathbf{1}_2$	$\mathbf{2}$	$\mathbf{3}_1$	$\mathbf{1}_2$
Z_3	ω	ω^2	ω^2	ω^2	1	1	1	1	1	ω^2	ω^2	ω^2
Z_4	1	i	-1	- i	1	1	i	i	1	1	1	-1

The superpotential of the model in the lepton sector

reads as follows

$$\begin{aligned}
w_l = & \frac{ye1}{\Lambda^3} e^c(l\varphi)_{11}(\varphi\varphi)_{11}h_d + \frac{ye2}{\Lambda^3} e^c((l\varphi)_2(\varphi\varphi)_{21})_{11}h_d + \frac{ye3}{\Lambda^3} e^c((l\varphi)_{31}(\varphi\varphi)_{31})_{11}h_d + \frac{ye4}{\Lambda^3} e^c((l\chi)_2(\chi\chi)_{21})_{11}h_d \\
& + \frac{ye5}{\Lambda^3} e^c((l\chi)_{31}(\chi\chi)_{31})_{11}h_d + \frac{ye6}{\Lambda^3} e^c(l\varphi)_{11}(\chi\chi)_{11}h_d + \frac{ye7}{\Lambda^3} e^c((l\varphi)_2(\chi\chi)_{21})_{11}h_d + \frac{ye8}{\Lambda^3} e^c((l\varphi)_{31}(\chi\chi)_{31})_{11}h_d \\
& + \frac{ye9}{\Lambda^3} e^c((l\chi)_2(\varphi\varphi)_{21})_{11}h_d + \frac{ye10}{\Lambda^3} e^c((l\chi)_{31}(\varphi\varphi)_{31})_{11}h_d + \frac{y\mu}{2\Lambda^2} \mu^c(l(\varphi\chi)_{32})_{12}h_d + \frac{y\tau}{\Lambda} \tau^c(l\varphi)_{11}h_d + \dots
\end{aligned} \quad (2)$$

$$w_\nu = \frac{y\nu1}{\Lambda} ((\nu^c l)_2 \eta)_{11} h_u + \frac{y\nu2}{\Lambda} ((\nu^c l)_3 \phi)_{11} h_u + \frac{1}{2} M (\nu^c \nu^c)_{11} + \dots, \quad (3)$$

where the dots denote higher order contributions.

The vacuum configuration can be determined from the vanishing of the derivatives of the superpotentials w_l and w_ν with respect to each component of the driving fields, as shown in [7]. Using this way, we can obtain the alignment of the VEVs of flavons as follows;

$$\begin{aligned}
\langle \varphi \rangle &= (0, v_\varphi, 0), \quad \langle \chi \rangle = (0, v_\chi, 0), \quad \langle \vartheta \rangle = v_\vartheta, \\
\langle \eta \rangle &= (v_\eta, v_\eta), \quad \langle \phi \rangle = (v_\phi, v_\phi, v_\phi), \quad \langle \Delta \rangle = v_\Delta.
\end{aligned} \quad (4)$$

With these VEVs alignments as well as breaking of electroweak gauge symmetry, the charged-lepton mass matrix is explicitly expressed as

$$m_l = \text{Diag.} \left(y_e \frac{v_\varphi^3}{\Lambda^3}, y_\mu \frac{v_\varphi v_\chi}{\Lambda^2}, y_\tau \frac{v_\varphi}{\Lambda} \right) v_d, \quad (5)$$

where we assume all components are real, and the neutrino sector gives rise to the following Dirac and Majorana matrices

$$\begin{aligned}
m_\nu^d &= \begin{pmatrix} 2be^{i\alpha_2} & ae^{i\alpha_1} - be^{i\alpha_2} & ae^{i\alpha_1} - be^{i\alpha_2} \\ ae^{i\alpha_1} - be^{i\alpha_2} & ae^{i\alpha_1} + 2be^{i\alpha_2} & -be^{i\alpha_2} \\ ae^{i\alpha_1} - be^{i\alpha_2} & -be^{i\alpha_2} & ae^{i\alpha_1} + 2be^{i\alpha_2} \end{pmatrix} v_u \\
&= e^{i\alpha_1} \begin{pmatrix} 2be^{i\phi} & a - be^{i\phi} & a - be^{i\phi} \\ a - be^{i\phi} & a + 2be^{i\phi} & -be^{i\phi} \\ a - be^{i\phi} & -be^{i\phi} & a + 2be^{i\phi} \end{pmatrix} v_u, \quad (6) \\
M_R &= \begin{pmatrix} M & 0 & 0 \\ 0 & 0 & M \\ 0 & M & 0 \end{pmatrix}, \quad (7)
\end{aligned}$$

where we assume the quantity M , a and b are real and positive quantities, and the relative phase $\phi \equiv \alpha_2 - \alpha_1$ is the only physical phase because the phase α_1 can be rotated away. After seesawing, the effective light neutrino mass matrix is obtained from seesaw formula $m_{\text{eff}} = -(m_\nu^d)^T M_R^{-1} m_\nu^d$, which can be diagonalized by the TBM mixing matrix as follows;

$$U_\nu^T m_{\text{eff}} U_\nu = \text{Diag.}(m_1, m_2, m_3), \quad (8)$$

where the mass eigenvalues are given as

$$\begin{aligned}
m_1 &= m_0(1 + 9r^2 - 6r \cos \phi), \\
m_2 &= 4m_0, \\
m_3 &= m_0(1 + 9r^2 + 6r \cos \phi),
\end{aligned} \quad (9)$$

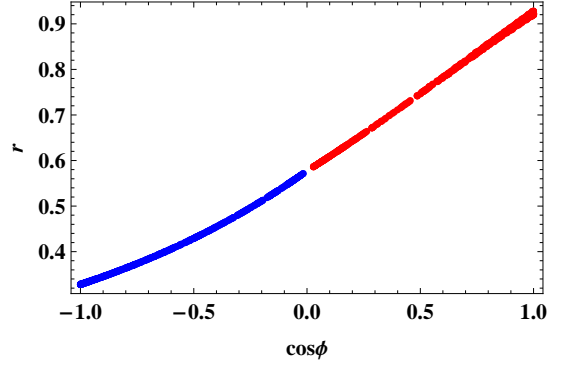


FIG. 1: Allowed parameter region of the ratio $r = b/a$ as a function of $\cos \phi$ constrained by the 1σ experimental data in Eq. (13). The thickness of the line reflects 1σ uncertainty due to experimental data. Here, the blue (dark) and red (light) curves correspond to the inverted and normal mass ordering of light neutrino, respectively.

with $r = b/a$, and $m_0 = v_u^2 a^2 / M$ where $v_u = v \sin \beta$ and $v = 176$ GeV. And the lepton mixing at low energy $U_{\text{PMNS}} = U_\nu$ is given by

$$U_{\text{PMNS}} = e^{-i\gamma_1/2} U_{\text{TBM}} \text{Diag.}(1, e^{i\beta_1}, e^{i\beta_2}), \quad (10)$$

where $\beta_1 = \gamma_1/2$, $\beta_2 = (\gamma_1 - \gamma_2)/2$ are the Majorana CP-violating phases, with

$$\begin{aligned}
\gamma_1 &= \arg\{(a - 3be^{i\phi})^2\}, \\
\gamma_2 &= \arg\{-(a + 3be^{i\phi})^2\}.
\end{aligned} \quad (11)$$

The phase factored out to the left have no physical meaning, since it can be eliminated by a redefinition of the charged lepton fields. There are the nine physical quantities consisting of the three light neutrino masses, the three mixing angles and the three CP-violating phases. The mixing angles are entirely fixed by the G_f symmetry group, predicting TBM and in turn no Dirac CP-violating phase, and the remaining 5 physical quantities $\beta_1, \beta_2, m_1, m_2$ and m_3 , are determined in terms of the five real parameters M, v_u, a, b and ϕ .

Because of the requirement of MSW resonance for solar neutrinos implying $\Delta m_{21}^2 > 0$, and the observed hierarchy $|\Delta m_{31}^2| \gg \Delta m_{21}^2$, there are two possible neutrino mass ordering depending on the sign of $\cos \phi$: (i) $m_1 < m_2 < m_3$ (normal mass ordering) which corresponds to $\cos \phi > 0$ and (ii) $m_3 < m_1 < m_2$ (inverted

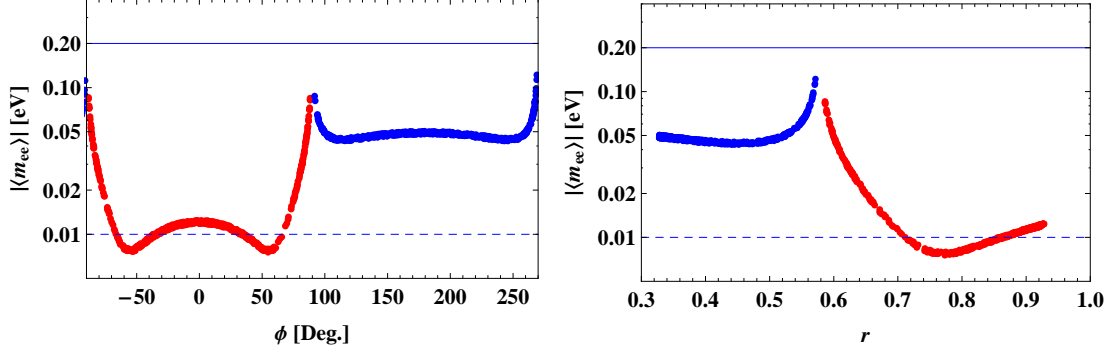


FIG. 2: Predictions of the effective mass $|\langle m_{ee} \rangle|$ for $0\nu 2\beta$ as a function of the phase ϕ in the left panel and the ratio r in the right panel based on the 1σ experimental results given in Eq. (13). Here, in both panels the red (light) and blue (dark) curves correspond to the normal mass spectrum of light neutrino and the inverted one, respectively.

mass ordering) which corresponds to $\cos \phi < 0$. The solar and atmospheric mass-squared differences, which are given by

$$\begin{aligned} \Delta m_{21}^2 &= 3m_0^2(1 - 3r^2 + 2r \cos \phi)(5 + 9r^2 - 6r \cos \phi), \\ |\Delta m_{31}^2| &= 24m_0^2 r |\cos \phi| (1 + 9r^2), \end{aligned} \quad (12)$$

are constrained by the neutrino oscillation experiments. Since neutrino oscillation data indicate that Δm_{21}^2 is positive, $1 - 3r^2 + 2r \cos \phi > 0$. It is interesting to see how the ratio $\Delta m_{21}^2 / \Delta m_{31}^2$ leads to the correlation between the parameters r and $\cos \phi$. From Eq.(12), we see that the ratio $\Delta m_{21}^2 / \Delta m_{31}^2$ is independent of m_0 , and that $\cos \phi = 0$ is not allowed, which is reflected in Fig. 1. For our purpose, we consider the experimental data at 1σ [1]:

$$|\Delta m_{31}^2| = (2.29 - 2.52) \times 10^{-3} \text{eV}^2,$$

$$\Delta m_{21}^2 = (7.45 - 7.88) \times 10^{-5} \text{eV}^2. \quad (13)$$

Hereafter, we use the 1σ confidence level experimental values of low energy observables for our numerical calculations. Imposing the experimental results, we present the correlations between r and $\cos \phi$ for normal mass hierarchy (red-plot) and inverted one (blue plot) in Fig. 1. From Eqs. (12,13), we can also determine the value of m_0 as a function of $\cos \phi$. Thanks to the zero entry in U_{PMNS} , m_3 does not contribute to the effective neutrino mass and thus only the phase β_1 can contribute to the $0\nu 2\beta$ decay amplitude, which can be written as $|\langle m_{ee} \rangle| = |m_1|U_{e1}|^2 + m_2|U_{e2}|^2 + m_3|U_{e3}|^2$, where U_{ei} ($i=1,2,3$) are the components of U_{PMNS} . As a result, the effective mass governing the $0\nu 2\beta$ decay is given by [7]

$$\begin{aligned} |\langle m_{ee} \rangle| &= \frac{1}{3} |2m_1 e^{2i\beta_1} + m_2| \\ &= 2m_0 \sqrt{1 - 4r \cos \phi + 2r^2(2 + 3 \cos 2\phi) - 12r^3 \cos \phi + 9r^4}. \end{aligned} \quad (14)$$

The behavior of $|\langle m_{ee} \rangle|$ is plotted in Fig. 2 as a function of the phase ϕ (left panel) and the ratio r (right panel). Here we note that m_0 is related with other parameters through the seesaw formula $m_0 = a^2 v_u^2 / M$. For our numerical analysis, we take $m_0 = 0.0035 - 0.04$, $m_0 = 0.012 - 0.05$ for normal and inverted orderings, respectively, which are consistent with the experimental results given in Eq. (13). Numerically, our prediction is turned out to be $0.0076 \text{eV} \leq |\langle m_{ee} \rangle| \leq 0.10 \text{eV}$ for the normal mass ordering (red-plot) and $0.043 \text{eV} \leq |\langle m_{ee} \rangle| \leq 0.12 \text{eV}$ for the inverted mass ordering (blue-plot), where the upper limits come from the cosmological bound on neutrino mass scale. We note that the existence of the lower bound on the prediction of m_{ee} in the case of the nor-

mal mass ordering is due to $|\cos \phi| \leq 1$, which is easily understood from Eq. (9), as also discussed in [7]. In Fig. 2, the horizontal solid and dashed lines correspond to the current lower bound sensitivity (0.2 eV) [18] and the future lower bound sensitivity (10^{-2} eV) [19] of $0\nu 2\beta$ experiments, respectively. The thickness of the lines correspond to the 1σ allowed ranges due to experimental results.

Using Eq. (11) we can obtain the explicit correlation between the phase ϕ and the Majorana phase β_1

$$\sin 2\beta_1 = \frac{6r \sin \phi (3r \cos \phi - 1)}{1 - 6r \cos \phi + 9r^2}. \quad (15)$$

Note here that the size of $\tan 2\beta_1$ is constrained by Fig.

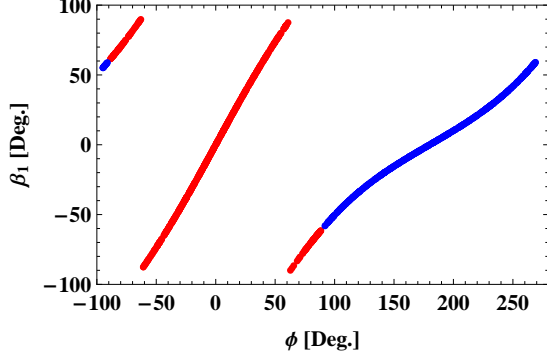


FIG. 3: Correlation of the Majorana CP phase β_1 with the phase ϕ constrained by the 1σ experimental data in Eq. (13). The red (light) and blue (dark) curves correspond to the normal mass spectrum of light neutrino and the inverted one, respectively.

3 represents the correlation between the phase ϕ and the Majorana phase β_1 for normal mass ordering (red-plot) and inverted one (blue-plot).

III. LEPTOGENESIS THROUGH SOFT S_4 BREAKING

In a basis where the charged current is flavor diagonal, the diagonalization of M_R leads to the mass eigenvalues

$$H = \begin{pmatrix} 2a^2 + 6b^2 - 4ab \cos \phi & \sqrt{2}(a^2 - 3b^2 + 2ab \cos \phi) & 0 \\ \sqrt{2}(a^2 - 3b^2 + 2ab \cos \phi) & 3a^2 + 3b^2 - 2ab \cos \phi & 0 \\ 0 & 0 & a^2 + 9b^2 + 6ab \cos \phi \end{pmatrix}. \quad (19)$$

We see that all $\text{Im}[H_{ij}]$ vanish and thus leptogenesis could not take place. However, when we consider low scale much below 10^9 GeV to avoid the abundance of gravitino overproduction problem [22] within a super-symmetric version, the τ and μ Yukawa couplings comes into equilibrium as the lepton asymmetry is being created. In that case, lepton number asymmetries and washout effects become flavor dependent, and this can give rise to a final baryon asymmetry which is different in size from the one generated if flavor effects are ignored [20, 21]. We call it flavored leptogenesis that occurs in general for $T \sim M_i(1 + \tan^2 \beta) \lesssim 10^9$ GeV in supersymmetric seesaw model [21]. In addition, since exact degenerate mass spectrum of the heavy RH neutrinos prevents leptogenesis to be occurred, in order for flavored leptogenesis to be viable, the degeneracy of the heavy RH neutrino masses should be lifted.

of heavy RH neutrinos given by

$$V_R^T M_R V_R = \text{Diag.}(M, M, -M), \quad (16)$$

where the diagonalizing matrix V_R is

$$V_R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}.$$

After performing a basis rotation so that the heavy RH Majorana mass matrix M_R becomes diagonal by the unitary matrix V_R , the Dirac mass matrix m_ν^d gets modified to

$$m_\nu^d \rightarrow Y_\nu v_u = V_R^T m_\nu^d \quad (17)$$

where the Yukawa coupling matrix Y_ν , is given as

$$Y_\nu = \begin{pmatrix} 2be^{i\phi} & a - be^{i\phi} & a - be^{i\phi} \\ \sqrt{2}(a - be^{i\phi}) & \frac{a+be^{i\phi}}{\sqrt{2}} & \frac{a+be^{i\phi}}{\sqrt{2}} \\ 0 & -\frac{a+3be^{i\phi}}{\sqrt{2}} & \frac{a+3be^{i\phi}}{\sqrt{2}} \end{pmatrix}. \quad (18)$$

Here, we notice that the CP phase ϕ existing in m_ν^d obviously gives rise to low-energy CP violation. On the other hand, leptogenesis is associated with both Y_ν itself and the combination of Yukawa coupling matrix, $H \equiv Y_\nu Y_\nu^\dagger$, which is given as

For an almost degenerate heavy Majorana neutrino mass spectrum, leptogenesis can be naturally implemented through the resonant-leptogenesis framework [13, 14, 23]. In this case, the CP asymmetry generated by the i -th heavy Majorana neutrino decaying into a lepton flavor α is given by [24]

$$\varepsilon_i^\alpha = \sum_{j \neq i} \frac{\text{Im}[H_{ij}(Y_\nu)_{i\alpha}(Y_\nu)_{j\alpha}^*]}{16\pi H_{ii}\delta_N^{ij}} \left(1 + \frac{\Gamma_j^2}{4M_j\delta_N^{ij2}}\right), \quad (20)$$

where $\Gamma_j = H_{jj}M_j/8\pi$ is the decay width of the j -th right-handed Majorana neutrino and δ_N^{ij} is the mass splitting parameter defined as

$$\delta_N^{ij} = 1 - \frac{M_j}{M_i}. \quad (21)$$

In order for resonant leptogenesis to be implemented successfully, soft terms of the form $\epsilon M \bar{\nu}_i^c \nu_j^c$ with small

dimensionless parameter ϵ [25] are added in Eq. (3), which lift the degeneracy of the heavy Majorana neutrino masses. Although there are several possibilities to incorporate the breaking parameter ϵ in M_R , which lead to the mass slitting parameter $|\delta_N^{ij}| \sim \epsilon$. Without loss of generality, we introduce a breaking term of the form $\epsilon M \bar{\nu}_2^c \nu_2^c$, which modifies the RH neutrino mass matrix as

$$M_R = \begin{pmatrix} M & 0 & 0 \\ 0 & \epsilon M & M \\ 0 & M & 0 \end{pmatrix}, \quad (22)$$

where the parameter ϵ is assumed to be real. M_R is diagonalized as $\tilde{V}_R^T M_R \tilde{V}_R = \text{Diag.}(M_1, M_2, M_3)$ with real eigenvalues given as

$$M_1 = M, \quad M_2 \simeq M \left(1 + \frac{\epsilon}{2}\right), \quad M_3 \simeq -M \left(1 - \frac{\epsilon}{2}\right) \quad (23)$$

and the diagonalizing matrix \tilde{V}_R is written as

$$\tilde{V}_R \simeq \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} + \frac{\epsilon}{4\sqrt{2}} & \frac{-1}{\sqrt{2}} + \frac{\epsilon}{4\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} - \frac{\epsilon}{4\sqrt{2}} & \frac{1}{\sqrt{2}} + \frac{\epsilon}{4\sqrt{2}} \end{pmatrix}. \quad (24)$$

In these expressions, the values of the parameters ϕ and r are determined from the analysis as demonstrated in sec.II, whereas ϵ and a are arbitrary. However, since the seesaw relation $a^2 = m_0 M / v_u^2$ as defined in Eq. (9), the value of a depends on the magnitude of M once m_0 is determined. Thus, in our numerical analysis, we take M and ϵ as input in the estimation of lepton asymmetry. Here, we note that although ϵ and M are taken to be independent parameters in our analysis, the predictions of the lepton asymmetries $\varepsilon_{1,2}^\alpha$ depends only on the ratio M/ϵ . We see from Eq. (27) that the asymmetries can be substantially enhanced by lowering ϵ . However, the value of ϵ is constrained by the validity of the perturbation. In order for the perturbative approach to be valid, the tree-level decay width Γ_i must be much smaller than the mass difference:

$$\Gamma_i = \frac{[Y_\nu Y_\nu^\dagger]_{ii}}{8\pi} M_i \ll M_2 - M_1 = \delta_N M_2, \quad i = 1, 2. \quad (28)$$

It is approximately given by

$$\Gamma_i \simeq a^2(1 + 3r^2 - 2r \cos \phi) / 4\pi \ll \delta_N^{21}.$$

Due to $a^2 = m_0 M / v_u^2$ and $\delta_N^{21} \simeq \epsilon/2$, if we take the seesaw scale, as an example, to be $M = 10^6$ GeV, the

In the basis where the heavy RH mass matrix real and diagonal, the Dirac Yukawa coupling matrix now reads as

$$\tilde{Y}_\nu = \frac{1}{v_u} \tilde{V}_R^T m_\nu^d, \quad (25)$$

where m_ν^d and \tilde{V}_R are given in Eq. (6) and Eq. (24), respectively. Assuming $\epsilon \ll 1$, we find that the matrix $\tilde{Y}_\nu \tilde{Y}_\nu^\dagger$ is real and almost the same as $\tilde{Y}_\nu \tilde{Y}_\nu^\dagger \simeq H$ given in Eq. (19). As a result, the contributions of N_3 to lepton asymmetries ε_i^α can be negligible, due to $H_{12(21)} \gg H_{13(31)} \simeq H_{23(32)} \simeq 0$. From the heavy Majorana neutrino masses given in Eq. (23) we can obtain the mass splitting parameter as follows,

$$\delta_N^{21} = -\delta_N^{12} \simeq \frac{\epsilon}{2}. \quad (26)$$

Then, combining with Eqs. (18, 19) and Eq. (26), the flavor dependent CP asymmetries ε_i^α can be obtained as follows

$$\begin{aligned} \varepsilon_1^e &\simeq -\frac{a^2(1 + 2r \cos \phi - 3r^2)}{\epsilon 4\pi(1 - 2r \cos \phi + 3r^2)} r \sin \phi, & \varepsilon_1^\mu &\simeq \varepsilon_1^\tau \simeq \frac{a^2(1 + 2r \cos \phi - 3r^2)}{\epsilon 8\pi(1 - 2r \cos \phi + 3r^2)} r \sin \phi \\ \varepsilon_2^e &\simeq -\frac{a^2(1 + 2r \cos \phi - 3r^2)}{\epsilon 2\pi(3 - 2r \cos \phi + 3r^2)} r \sin \phi, & \varepsilon_2^\mu &\simeq \varepsilon_2^\tau \simeq \frac{a^2(1 + 2r \cos \phi - 3r^2)}{\epsilon 4\pi(3 - 2r \cos \phi + 3r^2)} r \sin \phi \end{aligned} \quad (27)$$

value of a is the order of $\mathcal{O}(10^{-5})$, which requires $\epsilon \gg 10^{-10}$ for $\delta_N \simeq \epsilon/2$.

Once the initial values of ε_i^α are fixed, the final result of η_B can be obtained by solving a set of flavor dependent Boltzmann equations including the decay, inverse decay, and scattering processes as well as the nonperturbative sphaleron interaction. In order to estimate the wash-out effects, we introduce the parameters K_i^α which are the wash-out factors due to the inverse decay of the Majorana neutrino N_i into the lepton flavor $\alpha (= e, \mu, \tau)$ [26]. The explicit form of K_i^α is given by

$$K_i^\alpha = \frac{\Gamma_i^\alpha}{H(M_i)} = (Y_\nu^\dagger)_{\alpha i} (Y_\nu)_{i\alpha} \frac{v_u^2}{m_* M_i} \quad (29)$$

where Γ_i^α is the partial decay width of N_i into lepton flavor α and Higgs scalars, $H(M_i) \simeq (4\pi^3 g_*/45)^{\frac{1}{2}} M_i^2 / M_{Pl}$ with the Planck mass $M_{Pl} = 1.22 \times 10^{19}$ GeV and the effective number of degrees of freedom $g_* \simeq 228.75$ is the Hubble parameter at temperature $T = M_i$, and the equilibrium neutrino mass $m_* \simeq 10^{-3}$. From Eqs. (18) and (29) we can obtain the washout parameters as follows

$$K_1^e \simeq 4r^2 \frac{m_0}{m_*},$$

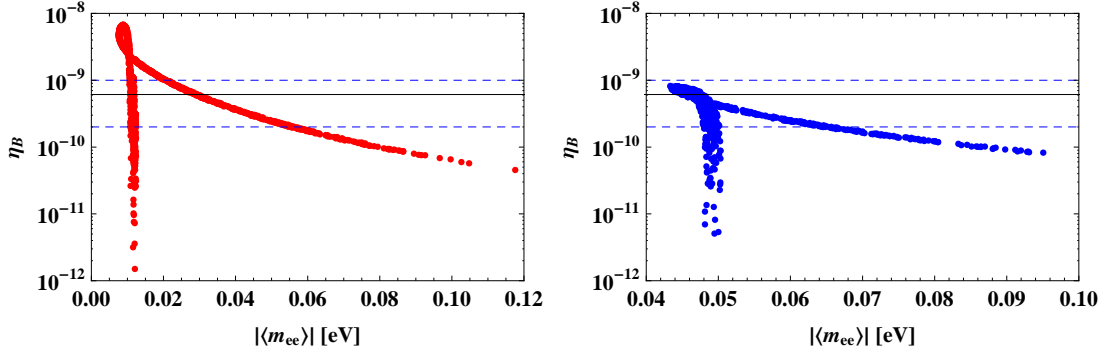


FIG. 4: The prediction of η_B as a function of $|\langle m_{ee} \rangle|$ for $M = 10^6$ GeV and the soft breaking parameter $\epsilon = 10^{-7}$. The left-plot and the right-plot represent the normal mass ordering of light neutrino and the inverted one, respectively. The solid horizontal line and the dotted horizontal lines corresponds to the experimental value of baryon asymmetry, $\eta_B^{\text{CMB}} = 6.1 \times 10^{-10}$, and phenomenologically allowed regions $2 \times 10^{-10} \leq \eta_B \leq 10^{-9}$, respectively.

$$\begin{aligned} K_1^{\mu,\tau} &\simeq \frac{m_0}{m_*} (1 - 2r \cos \phi + r^2), \\ K_2^e &\simeq \frac{2m_0}{m_*} (1 - 2r \cos \phi + r^2), \\ K_2^{\mu,\tau} &\simeq \frac{m_0}{2m_*} (1 + 2r \cos \phi + r^2). \end{aligned} \quad (30)$$

Since we take M to be 10^6 GeV, each lepton asymmetry for a single flavor in Eq. (27) is weighted differently by the corresponding washout parameter given by Eq. (30), and appears with different weight in the final formula for the baryon asymmetry [21, 26] as follows;

$$\eta_B \simeq -10^{-2} \sum_{N_i} \left[\varepsilon_i^e \kappa \left(\frac{93}{110} K_i^e \right) + \varepsilon_i^\mu \kappa \left(\frac{19}{30} K_i^e \right) + \varepsilon_i^\tau \kappa \left(\frac{19}{30} K_i^e \right) \right], \quad (31)$$

with wash-out factor

$$\kappa_i^\alpha \simeq \left(\frac{8.25}{K_i^\alpha} + \left(\frac{K_i^\alpha}{0.2} \right)^{1.16} \right)^{-1}. \quad (32)$$

As can be seen from Eqs. (27,30), since the lepton asymmetries in μ and τ flavors are equal to the first order, satisfying $\varepsilon_{1(2)}^\mu + \varepsilon_{1(2)}^\tau = -\varepsilon_{1(2)}^e$, and the washout factors in μ and τ are also equal, the value of baryon asymmetry can be obtained as

$$\eta_B \simeq 10^{-2} \{ \varepsilon_1^e (\kappa_1^\mu - \kappa_1^e) + \varepsilon_2^e (\kappa_2^\mu - \kappa_2^e) \}. \quad (33)$$

Numerically, the washout factors for normal mass spectrum of neutrino and the inverted one are given as

$$\begin{aligned} |\kappa_1^e - \kappa_1^\mu| &\lesssim 0.06, \quad |\kappa_2^\mu - \kappa_2^e| \lesssim 0.05, \\ \kappa_1^e - \kappa_1^\mu &\lesssim 0.02, \quad \kappa_2^\mu - \kappa_2^e \lesssim 0.05, \end{aligned} \quad (34)$$

respectively. Thus, taking Eqs. (33,34) into account, $|\varepsilon_{1(2)}^e| \sim 10^{-6-7}$ is needed to obtain a successful leptogenesis, which in turn means that the value of soft breaking parameter $\epsilon \lesssim 10^{-6}$ is required for $M = 10^6$ GeV.

Before going to discuss the value of η_B , we consider the effects of soft breaking term on the light neutrino observables. The effective neutrino mass matrix is modified

due to the soft breaking term and thus it can be diagonalized as $\tilde{U}_\nu^T \tilde{m}_{\text{eff}} \tilde{U}_\nu = \text{Diag.}(\tilde{m}_1, \tilde{m}_2, \tilde{m}_3)$ with the real eigenvalues

$$\tilde{m}_i = m_i + m_i \mathcal{O}(\epsilon), \quad (35)$$

and the mixing matrix $\tilde{U}_\nu = U_{\text{PMNS}}$ can be written as

$$U_{\text{PMNS}} = U_{TB} P_\nu + \delta U(\epsilon) P_\nu \quad (36)$$

where $P_\nu = \text{Diag.}(1, e^{i\beta_1}, e^{i\beta_2})$ is diagonal matrix of Majorana phases defined in Eq. (10). Note here that, since ϵ is real, the Majorana CP phases $\beta_{1,2}$ are not affected from the soft breaking. However, it is obvious that the effects of ϵ to the light neutrino mass eigenvalues and the neutrino mixing angles are negligible due to $\epsilon \lesssim 10^{-6}$. As expected, the value of $|\langle m_{ee} \rangle|$ is almost the same as Eq. (14).

The predictions for η_B as a function of $|\langle m_{ee} \rangle|$ are shown in Fig. 4 where we have used $M = 10^6$ GeV, $\epsilon = 10^{-7}$ and $\tan \beta = 2.5$ as inputs. Please note that if there is a mass splitting of Majorana neutrinos by a renormalization group equation, the value of $\tan \beta$ can be crucial to have a successful leptogenesis. However, its value does not affect significantly our results. So, for simplicity, we take $\tan \beta = 2.5$. The horizontal solid and

dashed lines correspond to the central value of the experiment result of BAU $\eta_B^{\text{CMB}} = 6.1 \times 10^{-10}$ [27] and the phenomenologically allowed regions $2 \times 10^{-10} \leq \eta_B \leq 10^{-9}$, respectively. As shown in Fig. 4, the current observation of η_B^{CMB} can narrowly constrain the value of $|\langle m_{ee} \rangle|$ for the normal hierarchical mass spectrum of light neutrino and inverted one, respectively in the case that the scale of leptogenesis is $10^3 < M < 10^6$ GeV and ϵ is small enough. Combining the results presented in Figs. 2 and 3 with those from leptogenesis, we can pin down the Majorana CP phase β_1 via the parameter ϕ .

IV. CONCLUSIONS

We have examined how leptogenesis can be implemented in the seesaw model with S_4 flavor symmetry which leads to the neutrino TBM mixing matrix and degenerate RH neutrino spectrum. In order for leptogenesis to be viable, $Y_\nu Y_\nu^\dagger$ should contain nontrivial imaginary part and the degeneracy of heavy right-handed Majorana masses has to be lifted. In our study, we have shown that flavored resonant leptogenesis can be successfully realized by introducing a tiny soft S_4 symmetry breaking term in M_R , which can much lower the seesaw scale. Even though we can lower the seesaw scale down to TeV scale, it would be difficult to probe it directly in collider experiments because the couplings between the heavy neutrinos and the light neutrinos are so small in our scenario. Even though such a tiny soft breaking term is essential

for leptogenesis, it does not significantly affect the low energy observables. We have also investigated how the effective neutrino mass $|\langle m_{ee} \rangle|$ associated with $0\nu 2\beta$ decay can be predicted along with the light neutrino mass hierarchies in our scenario for leptogenesis by imposing experimental data of low-energy observables. Interestingly enough, we have found a direct link between leptogenesis and neutrinoless double beta decay characterized by $|\langle m_{ee} \rangle|$ through a high energy CP phase ϕ which is correlated with low energy Majorana CP phases. We also have shown that our predictions of $|\langle m_{ee} \rangle|$ for some fixed parameters can be constrained by the current observation of baryon asymmetry in the case that the scale of leptogenesis is $10^3 < M < 10^6$ GeV and ϵ is small enough.

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